Principles of Communications

ECS 332

Asst. Prof. Dr. Prapun Suksompong

prapun@siit.tu.ac.th

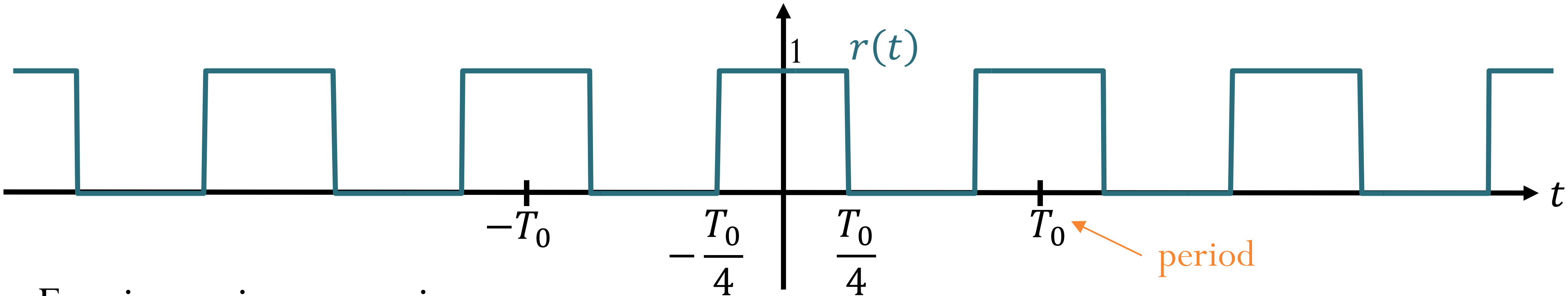
4.3 Fourier Series

Office Hours:

BKD, 6th floor of Sirindhralai building

Wednesday	14:00-15:30
Friday	14:00-15:30

Square Wave

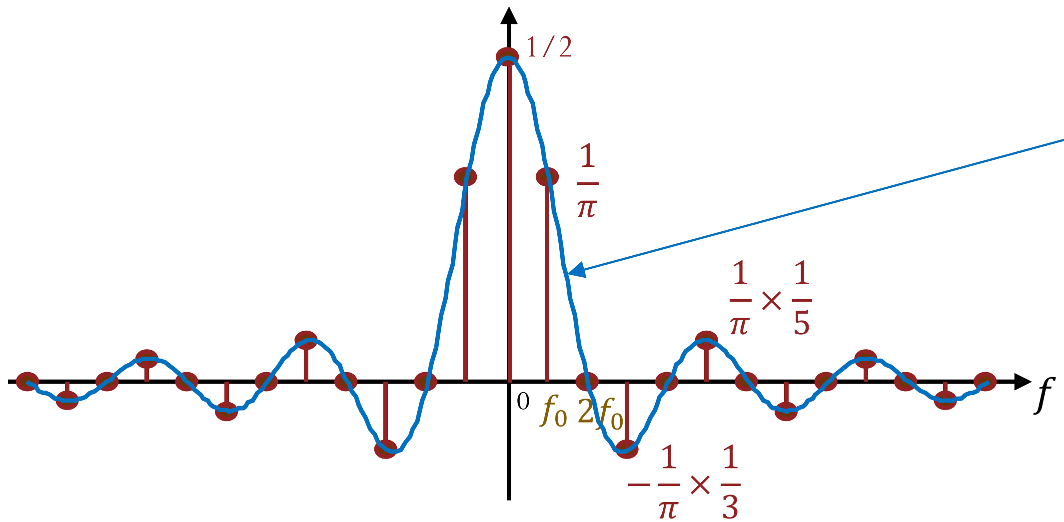


Fourier series expansion:

Fundamental frequency $= 1/T_0$

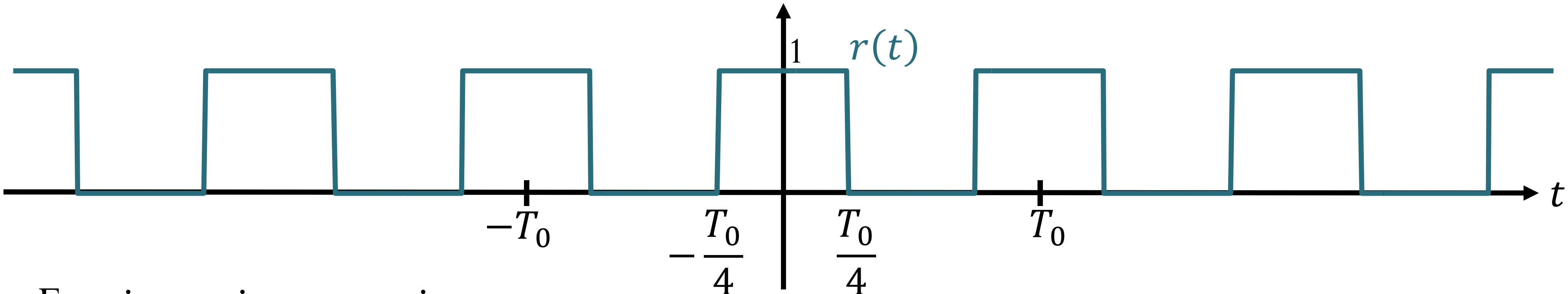
$$r(t) = \frac{1}{2} + \frac{1}{\pi}e^{j(2\pi f_0 t)} - \frac{1}{3\pi}e^{j(2\pi(3f_0)t)} + \frac{1}{5\pi}e^{j(2\pi(5f_0)t)} + \cdots$$

$$+ \frac{1}{\pi}e^{j(2\pi(-f_0)t)} - \frac{1}{3\pi}e^{j(2\pi(-3f_0)t)} + \frac{1}{5\pi}e^{j(2\pi(-5f_0)t)} + \cdots$$



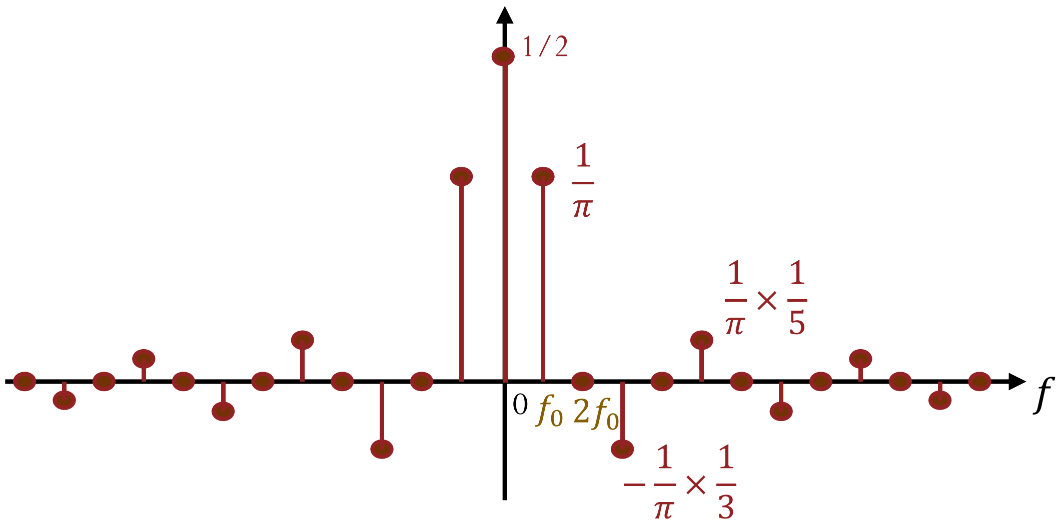
$\frac{1}{T_0}R_{T_0}(f)$: the **scaled Fourier transform** of the **restricted** (one period) version of $r(t)$.

Square Wave



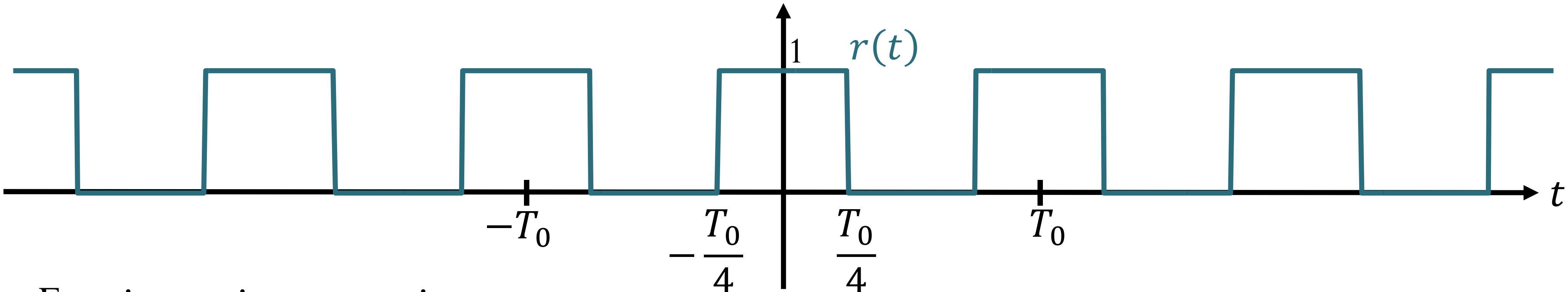
Fourier series expansion:

$$r(t) = \frac{1}{2} + \frac{1}{\pi} e^{j(2\pi f_0 t)} - \frac{1}{3\pi} e^{j(2\pi(3f_0)t)} + \frac{1}{5\pi} e^{j(2\pi(5f_0)t)} + \cdots$$
$$+ \frac{1}{\pi} e^{j(2\pi(-f_0)t)} - \frac{1}{3\pi} e^{j(2\pi(-3f_0)t)} + \frac{1}{5\pi} e^{j(2\pi(-5f_0)t)} + \cdots$$



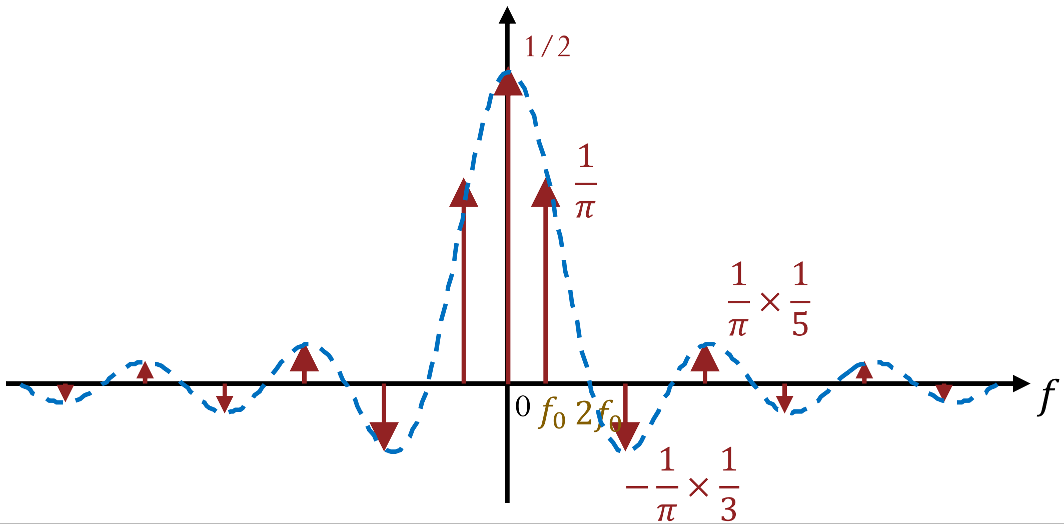
These "lines" are collectively referred to as the (two-sided) **line spectrum** of the periodic signal.

Square Wave



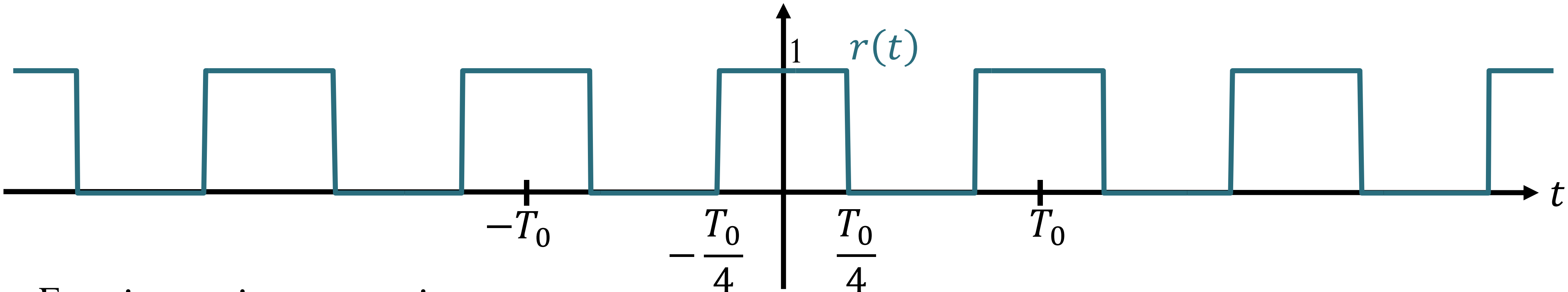
Fourier series expansion:

$$r(t) = \frac{1}{2} + \frac{1}{\pi}e^{j(2\pi f_0 t)} - \frac{1}{3\pi}e^{j(2\pi(3f_0)t)} + \frac{1}{5\pi}e^{j(2\pi(5f_0)t)} + \cdots$$
$$+ \frac{1}{\pi}e^{j(2\pi(-f_0)t)} - \frac{1}{3\pi}e^{j(2\pi(-3f_0)t)} + \frac{1}{5\pi}e^{j(2\pi(-5f_0)t)} + \cdots$$



Simply changing them to "arrows" (representing the delta functions). Collectively, they are now the **Fourier transform** of your periodic signal.

Square Wave



Fourier series expansion:

$$r(t) = \frac{1}{2} + \frac{1}{\pi} e^{j(2\pi f_0 t)} - \frac{1}{3\pi} e^{j(2\pi(3f_0)t)} + \frac{1}{5\pi} e^{j(2\pi(5f_0)t)} + \cdots$$

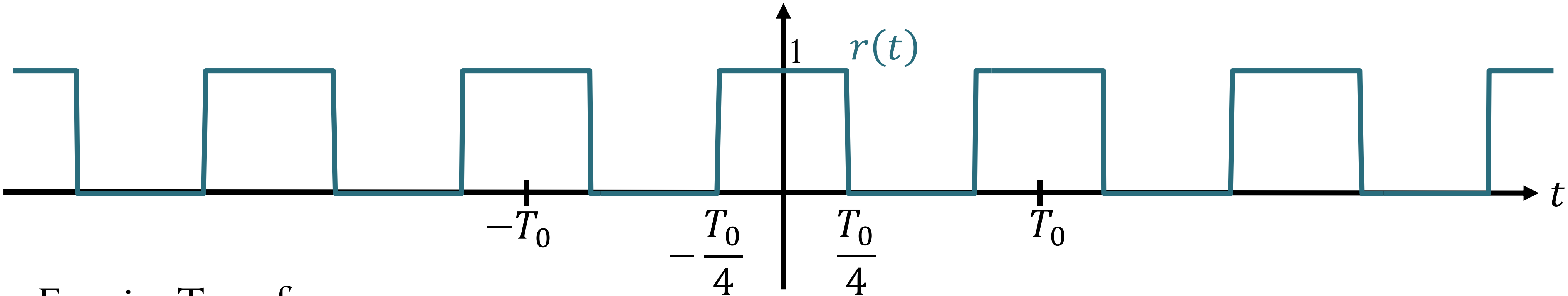
$$+ \frac{1}{\pi} e^{j(2\pi(-f_0)t)} - \frac{1}{3\pi} e^{j(2\pi(-3f_0)t)} + \frac{1}{5\pi} e^{j(2\pi(-5f_0)t)} + \cdots$$

$\mathscr{F}$ ↓

$$R(f) = \frac{1}{2} + \frac{1}{\pi}\delta(f - f_0) - \frac{1}{3\pi}\delta(f - 3f_0) + \frac{1}{5\pi}\delta(f - 5f_0) + \cdots$$

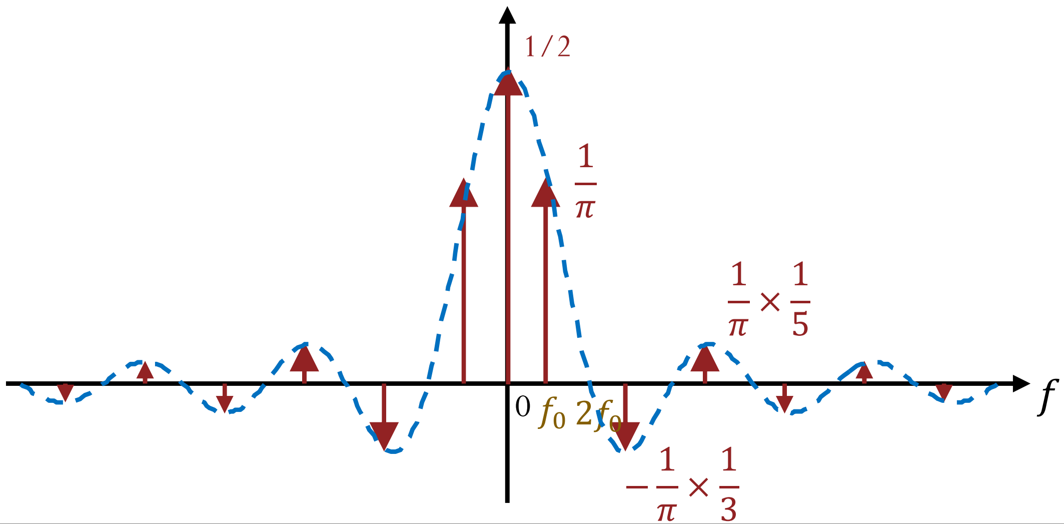
$$+ \frac{1}{\pi}\delta\big(f - (-f_0)\big) - \frac{1}{3\pi}\delta\big(f - (-3f_0)\big) + \frac{1}{5\pi}\delta\big(f - (-5f_0)\big) + \cdots$$

Fourier Transform of Square Wave



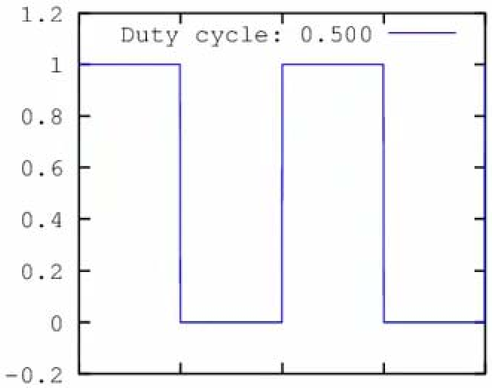
Fourier Transform:

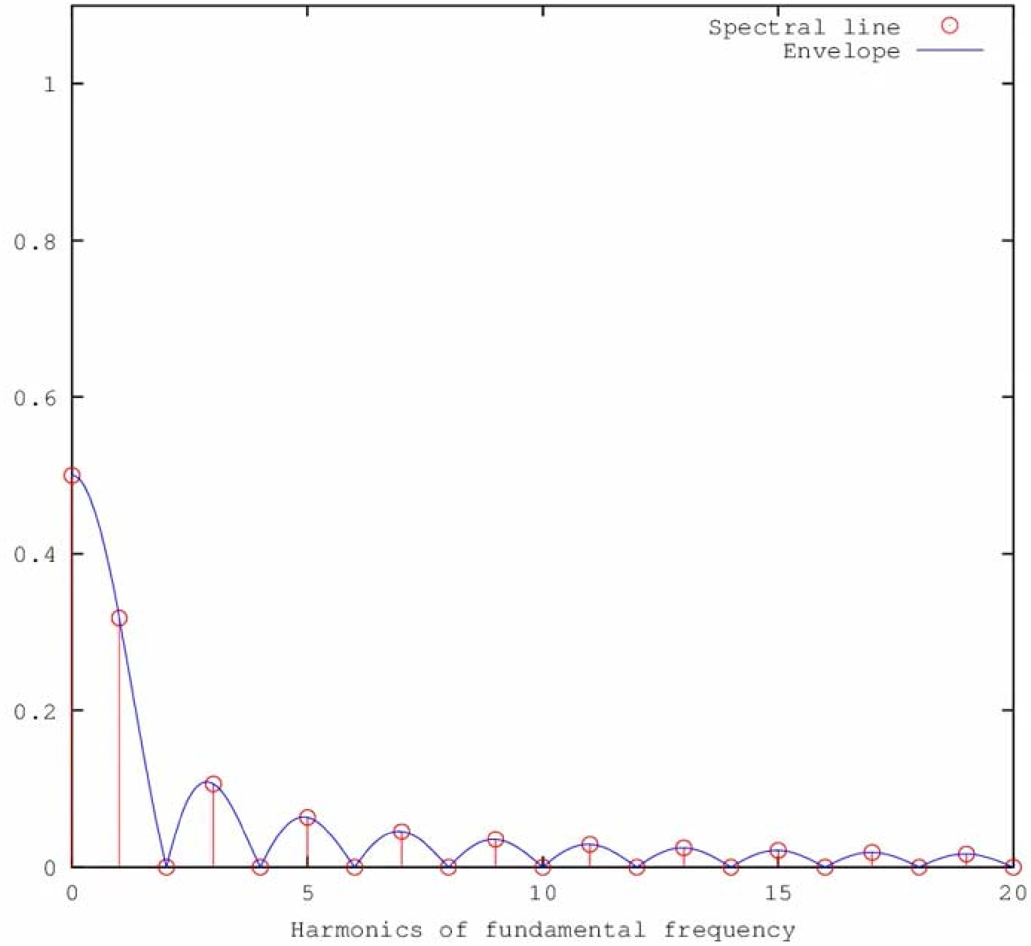
$$R(f) = \frac{1}{2} + \frac{1}{\pi}\delta(f - f_0) - \frac{1}{3\pi}\delta(f - 3f_0) + \frac{1}{5\pi}\delta(f - 5f_0) + \cdots$$
$$+ \frac{1}{\pi}\delta\big(f - (-f_0)\big) - \frac{1}{3\pi}\delta\big(f - (-3f_0)\big) + \frac{1}{5\pi}\delta\big(f - (-5f_0)\big) + \cdots$$



Effect of Duty Cycle

Duty cycle = 0.500

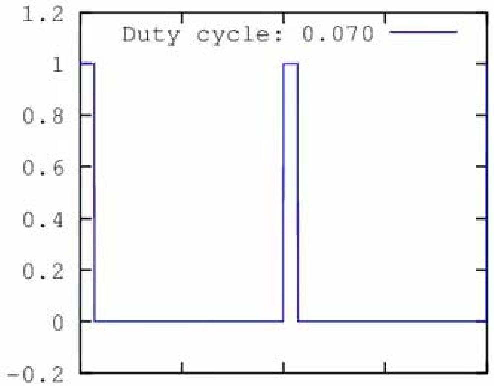


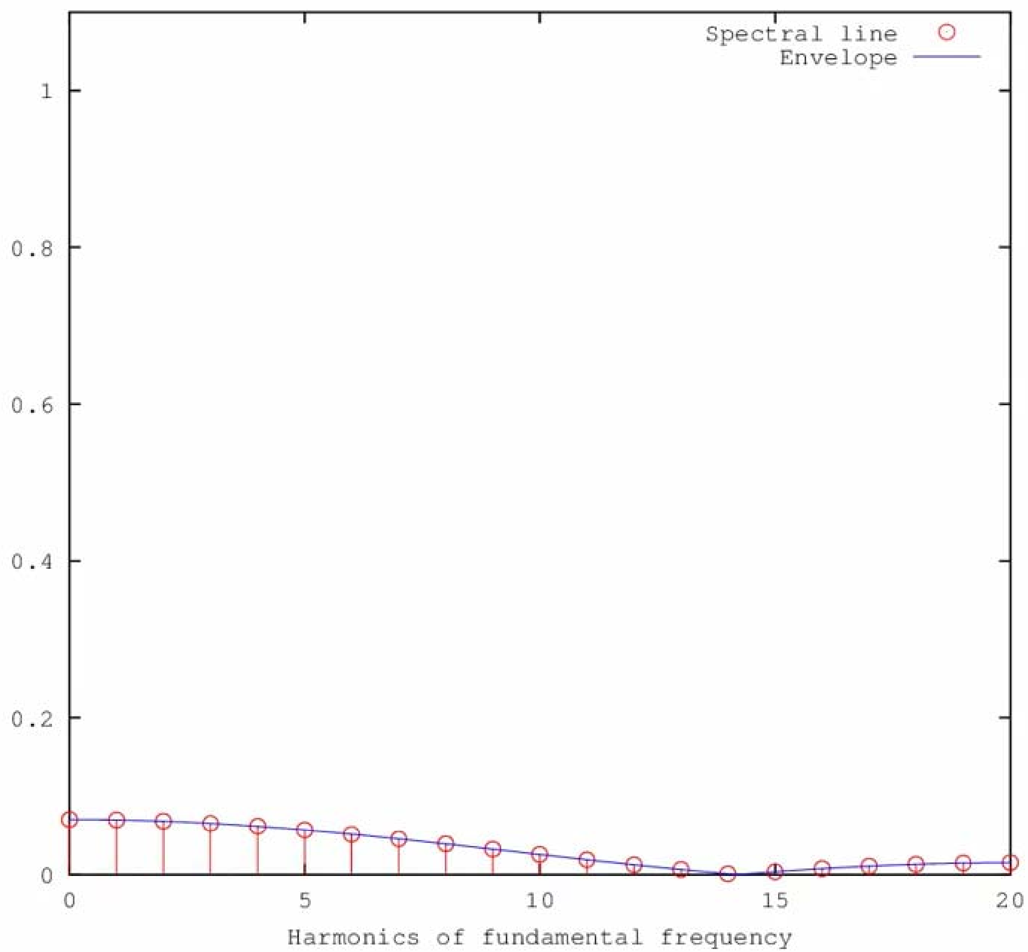


When duty cycle = 1/2, the **2nd** harmonic (along with its muliples) is suppressed.

Effect of Duty Cycle

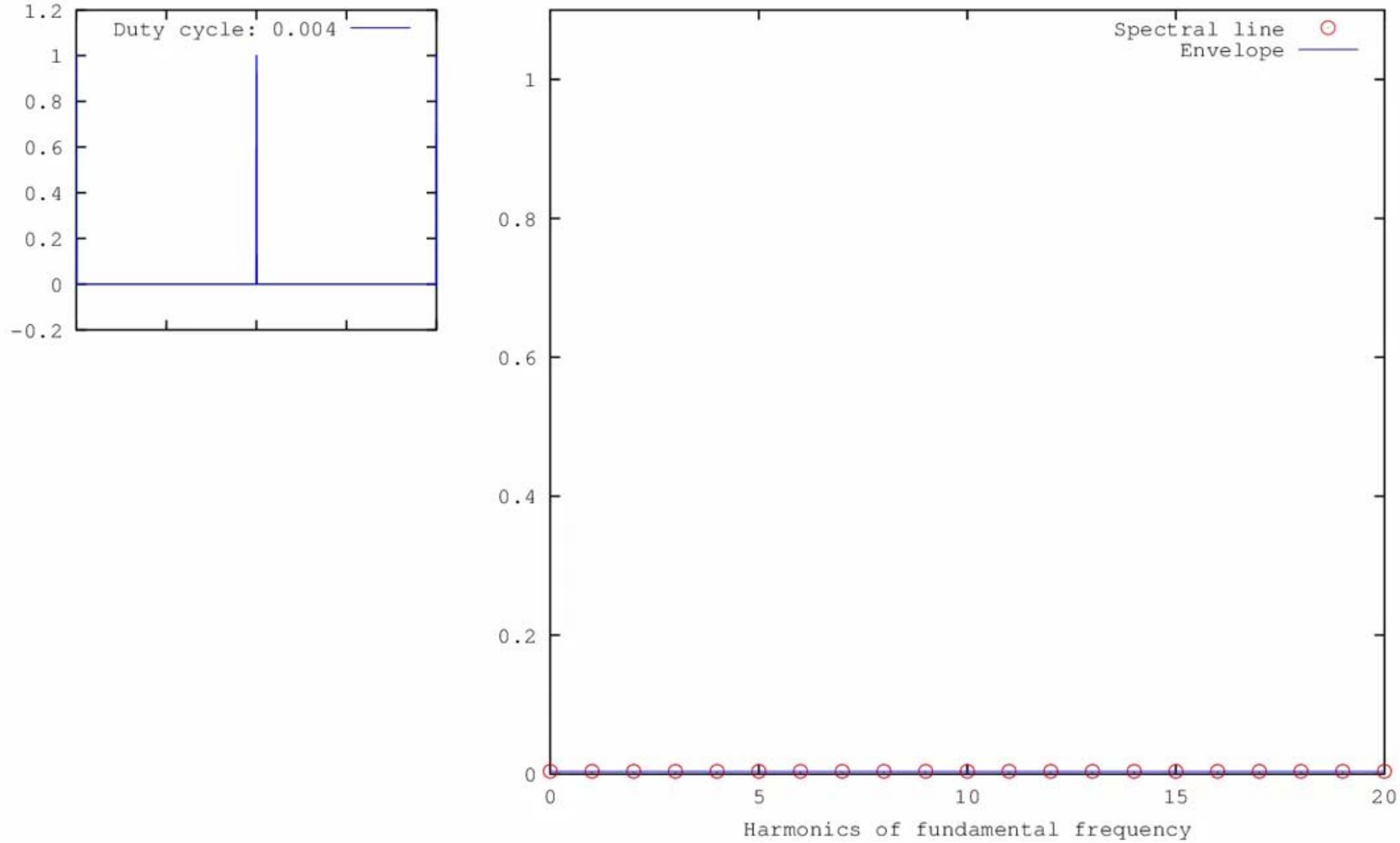
Duty cycle = 0.070





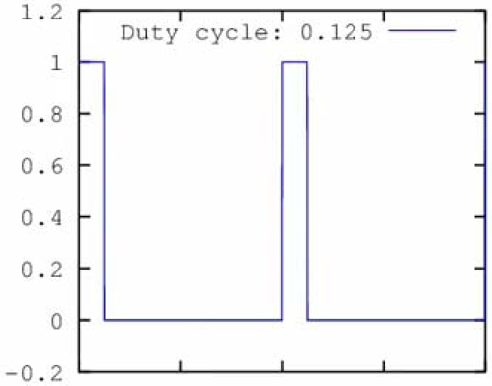
Note that it is not always the case that the 2nd harmonic (along with its muliples) is suppressed.

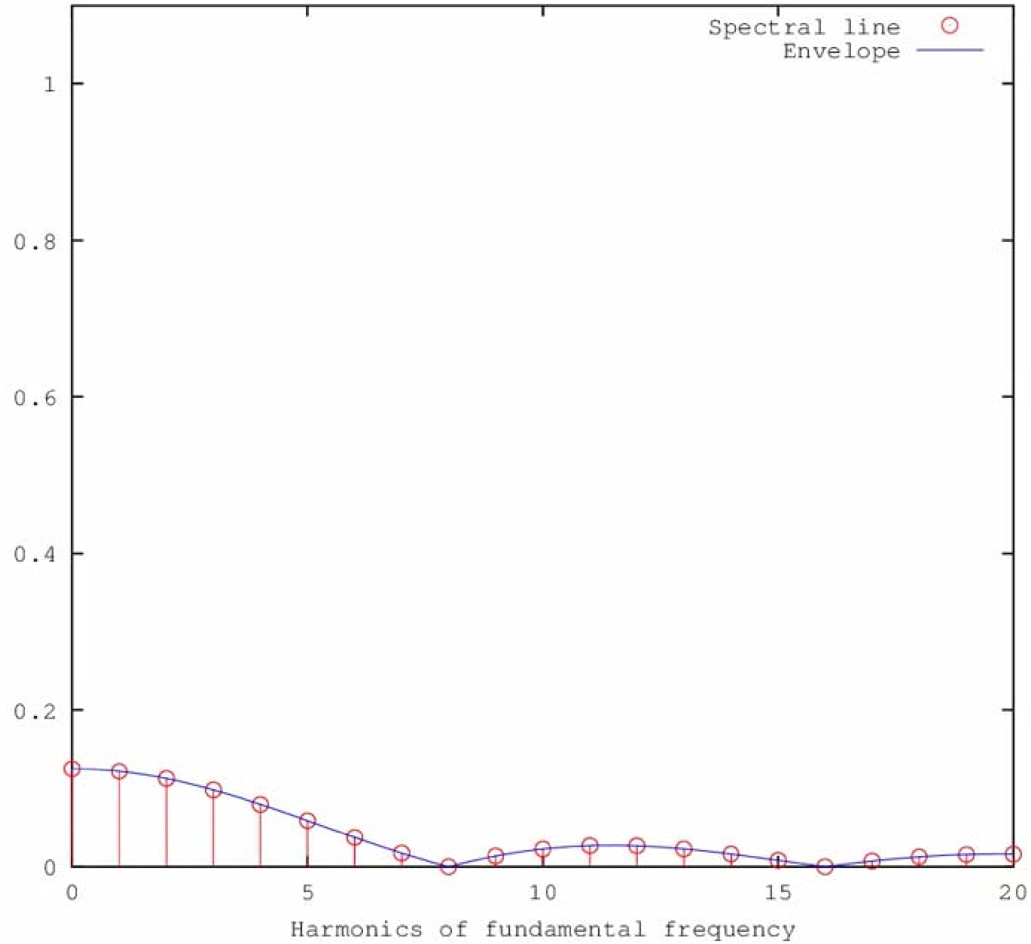
Effect of Duty Cycle



Effect of Duty Cycle

Duty cycle = 0.125

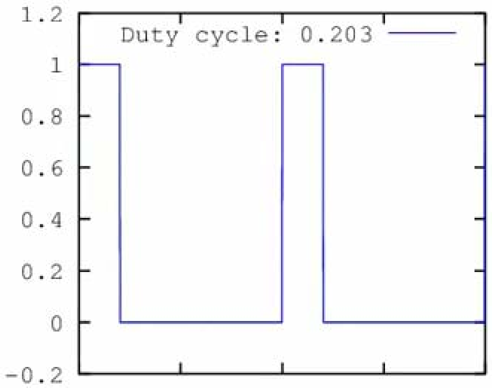


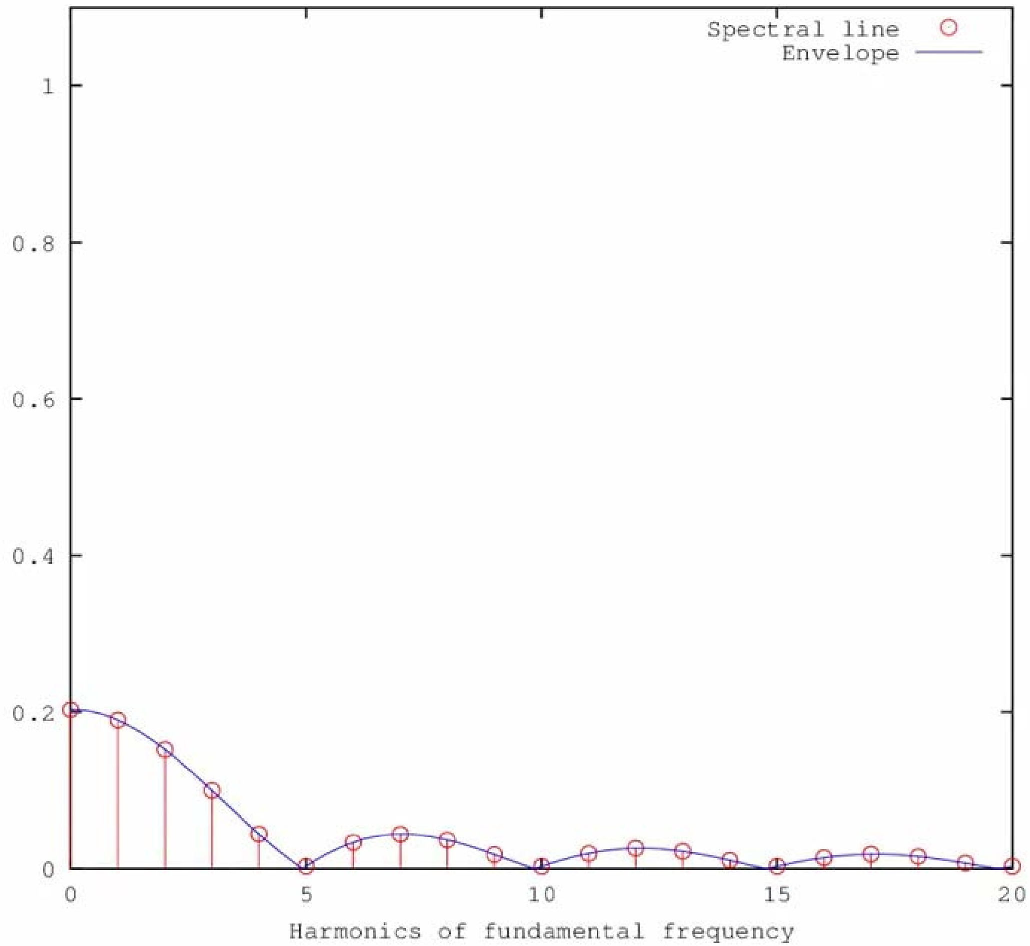


When duty cycle = 1/8, the **8th** harmonic (along with its muliples) is suppressed.

Effect of Duty Cycle

Duty cycle = 0.203

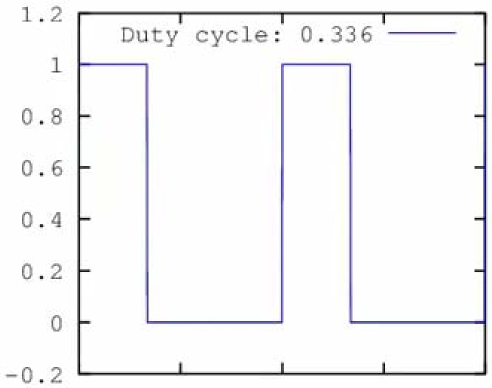


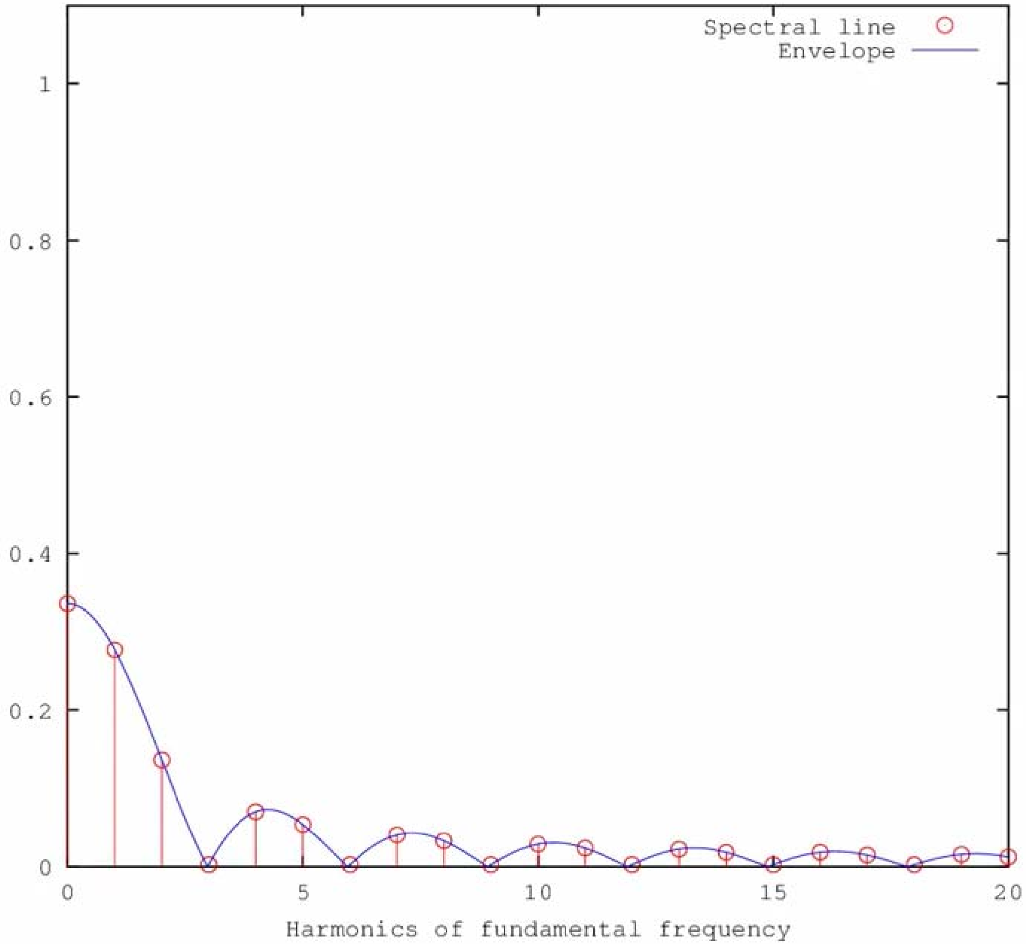


When duty cycle = 1/5, the **5th** harmonic (along with its muliples) is suppressed.

Effect of Duty Cycle

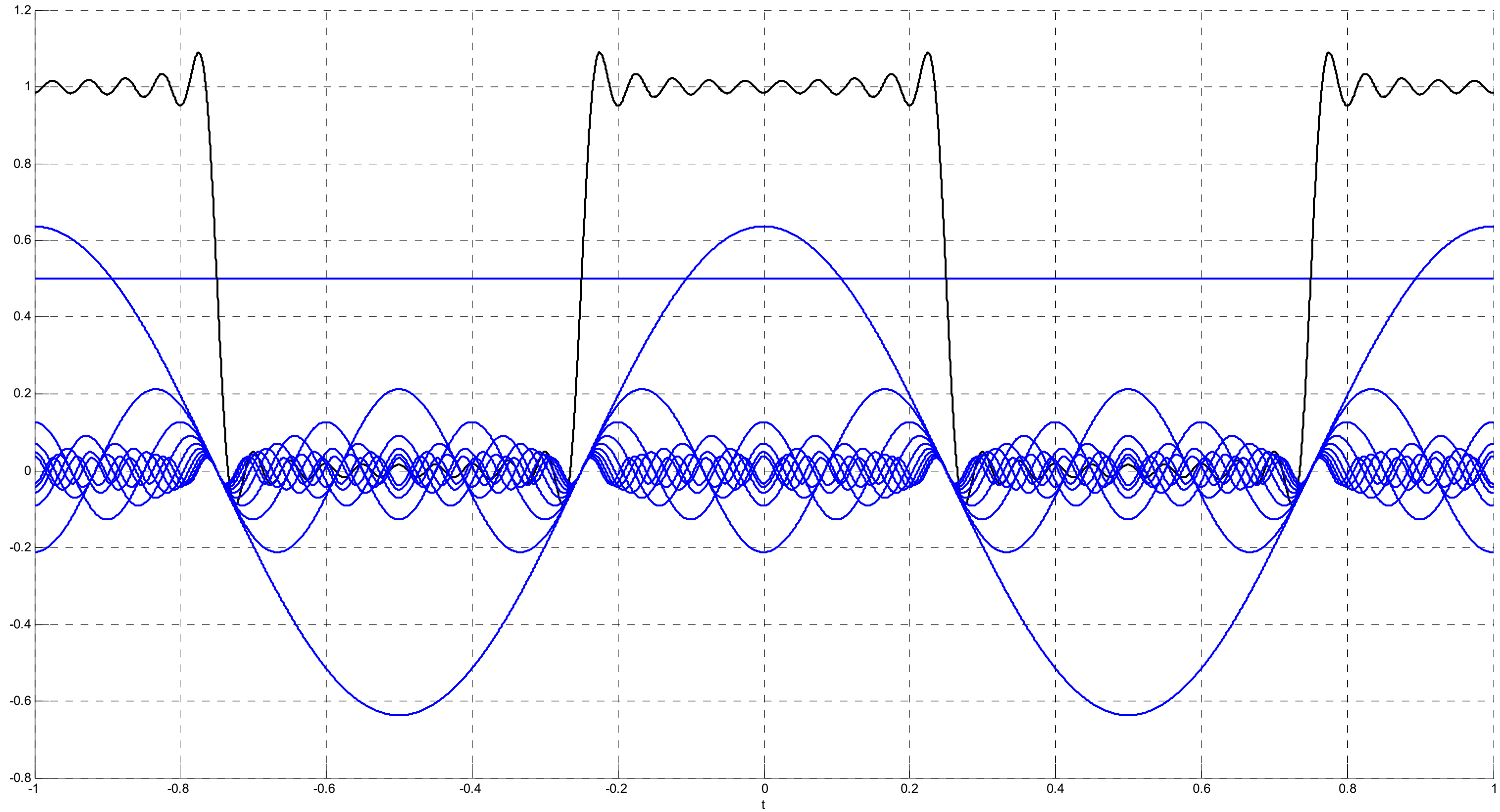
Duty cycle = 0.336





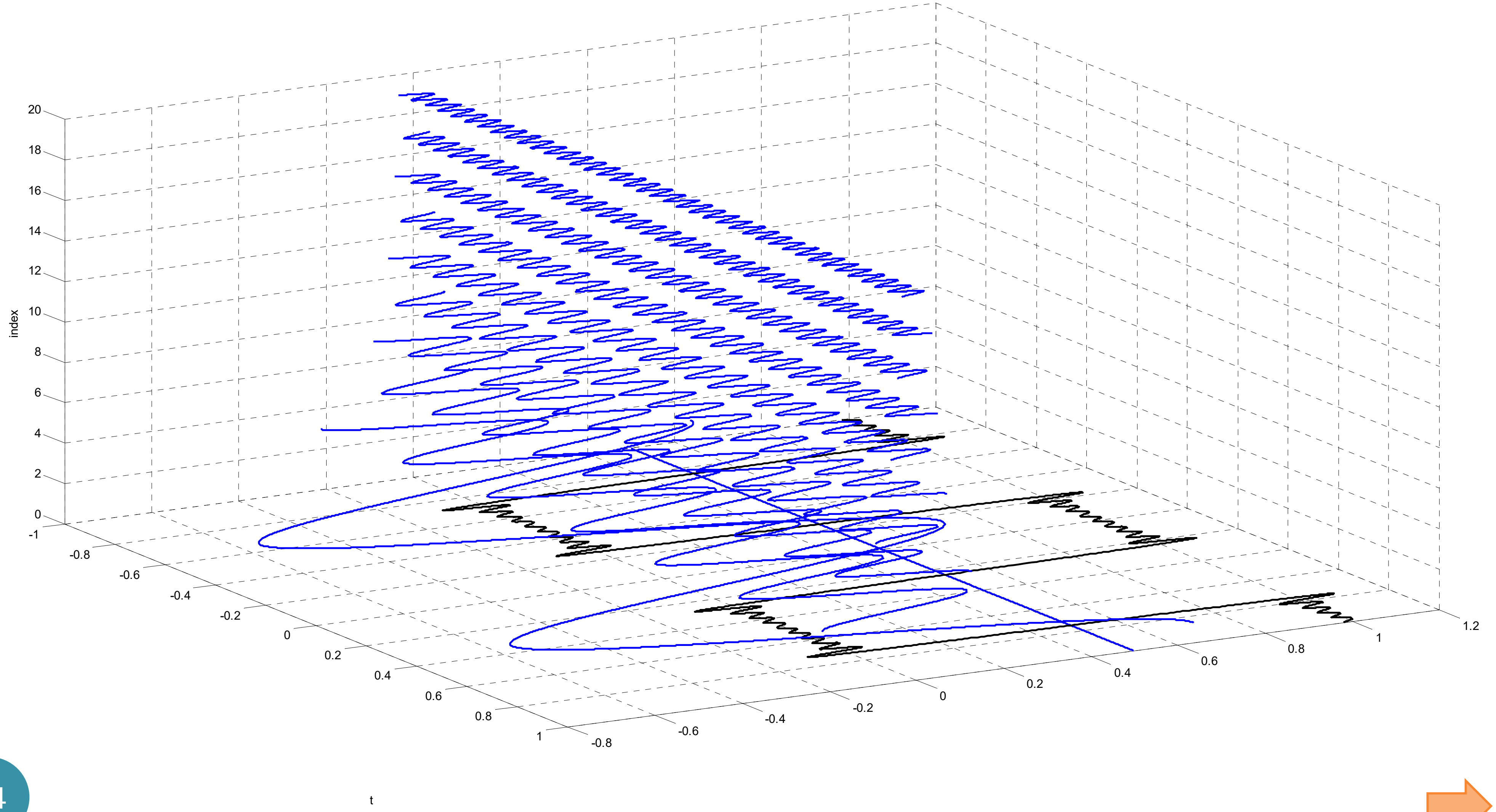
When duty cycle = 1/3, the **3rd** harmonic (along with its muliples) is suppressed.

Fourier Series: Ex 1

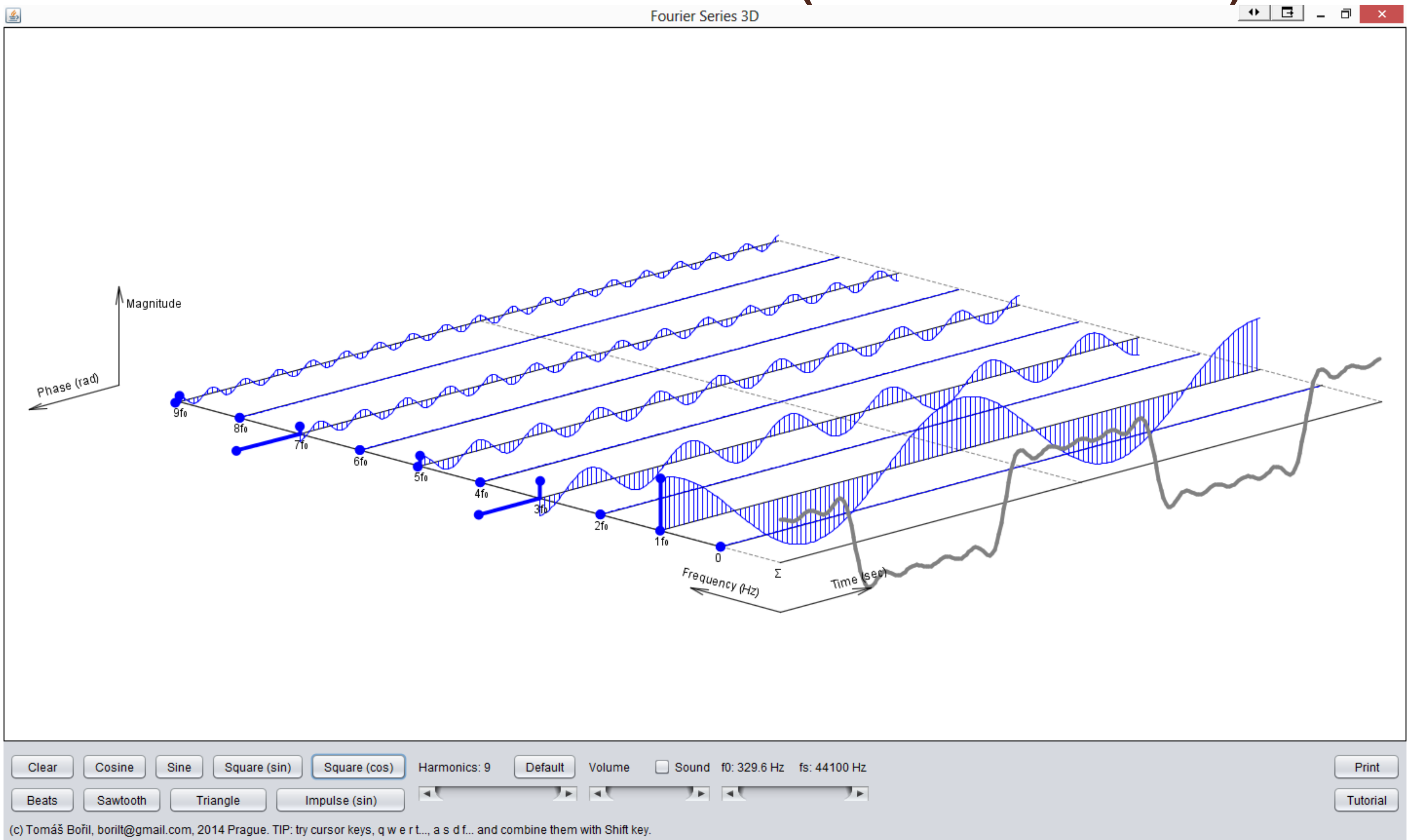


ECS332_4_Amplitude_Modulation_Fourier_Ex1.fig

Fourier Series: Ex 1



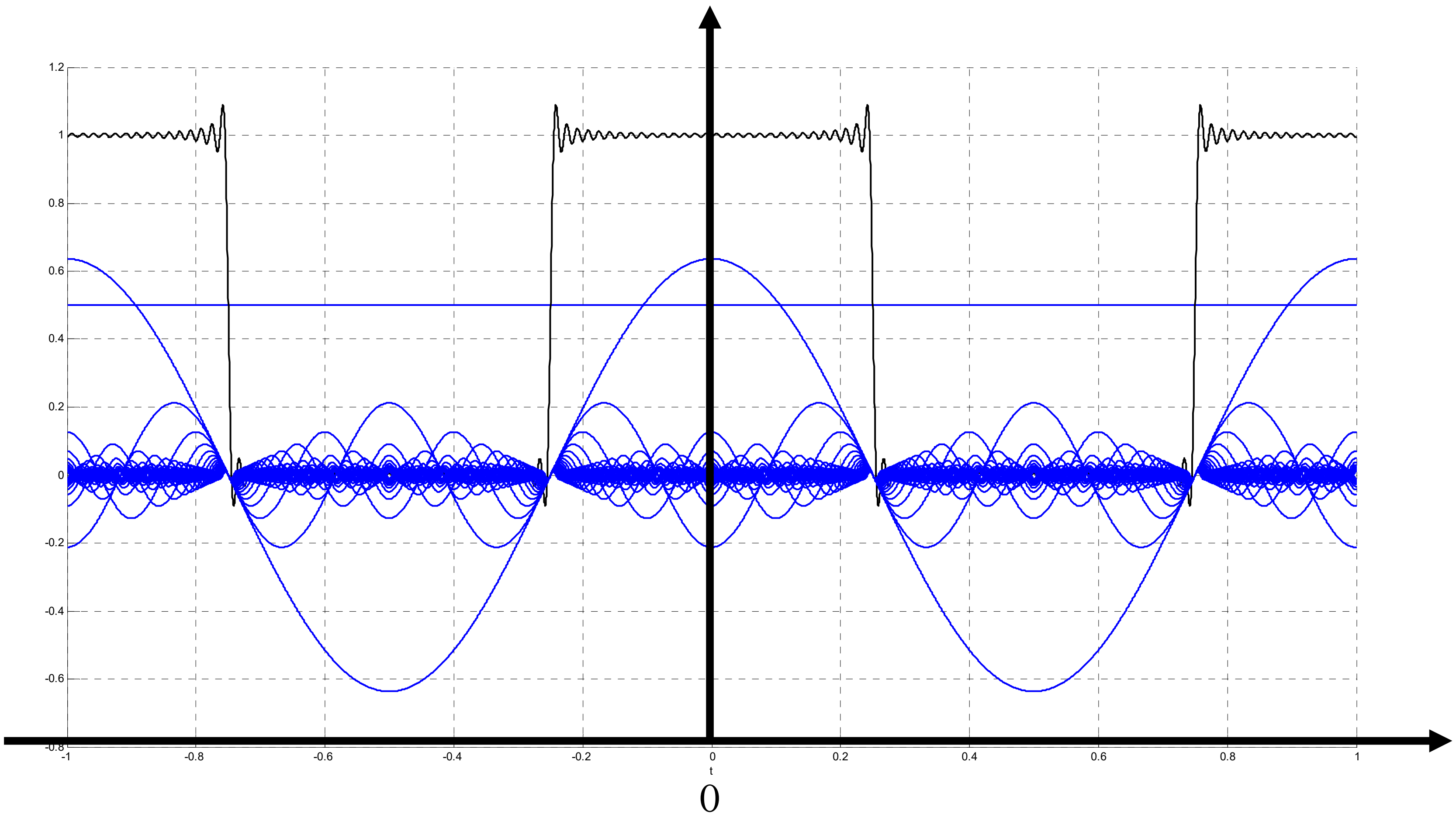
Fourier Series: Ex 1 (interactive)



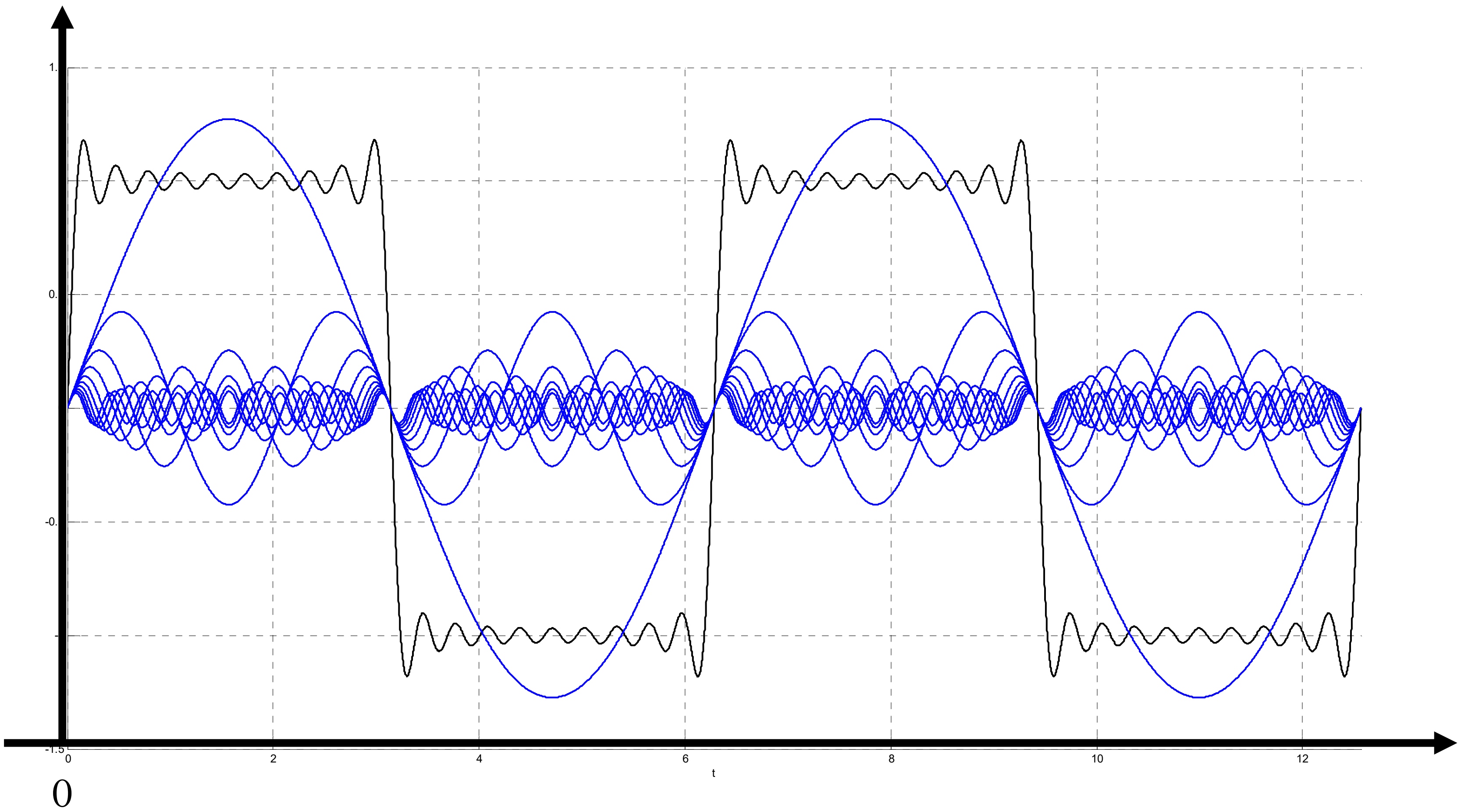
ECS332_4_Amplitude_Modulation_Fourier_Ex1.jar

[http://www.tomasboril.cz/hobbies_programs_en.html]

Fourier Series: Ex 1

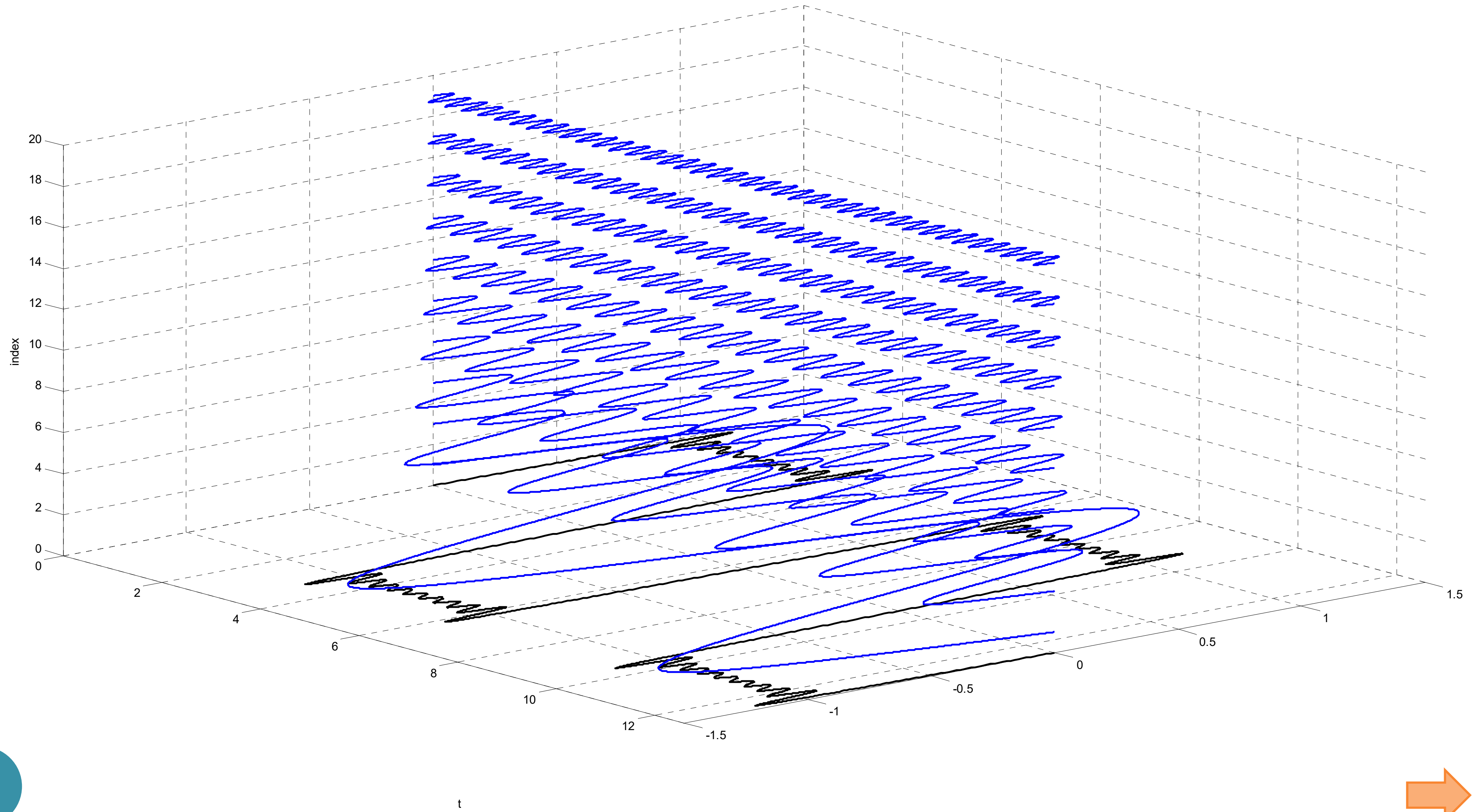


Fourier Series: Ex 2



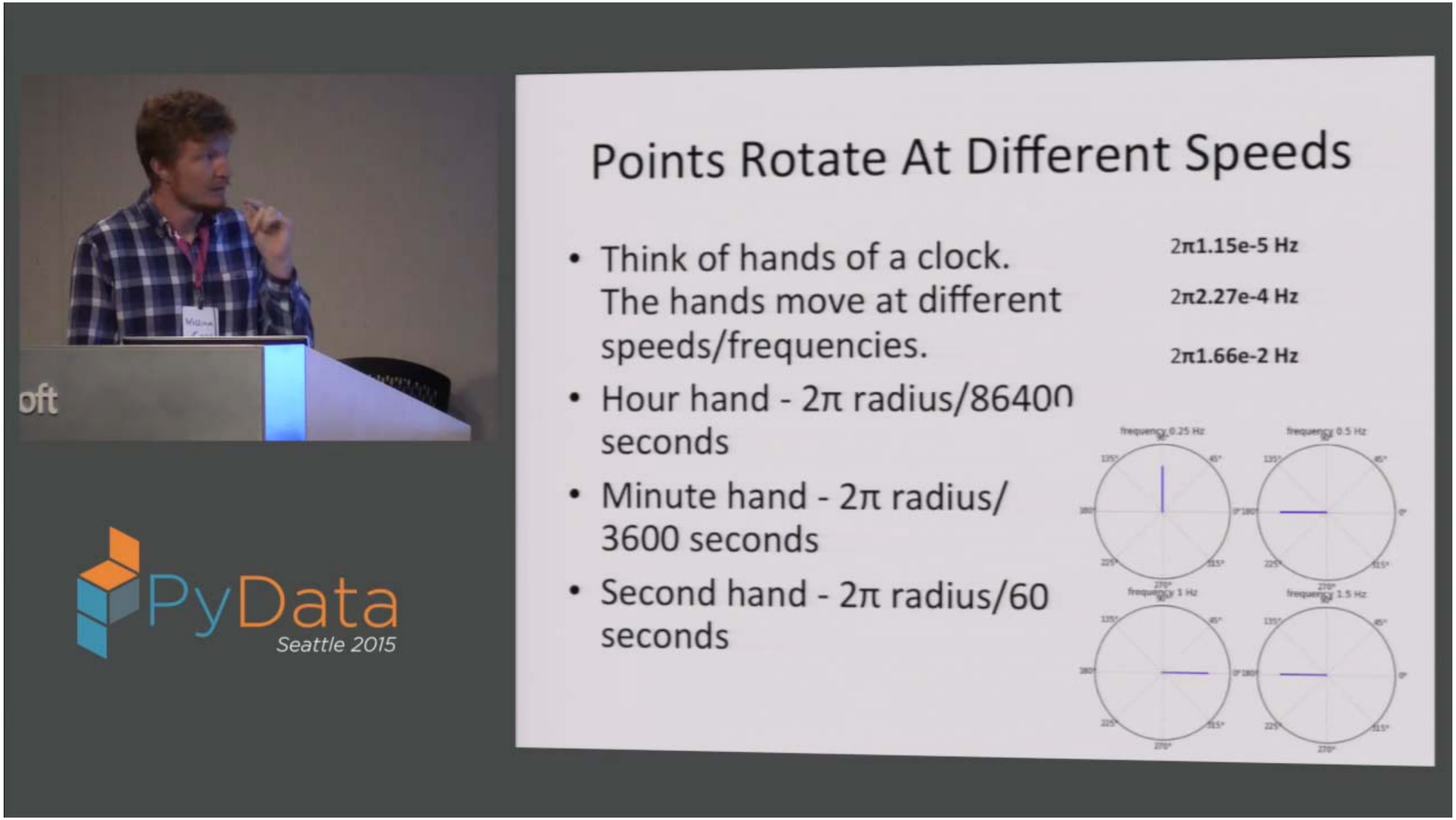
ECS332_4_Amplitude_Modulation_Fourier_Ex2.fig

Fourier Series: Ex 2

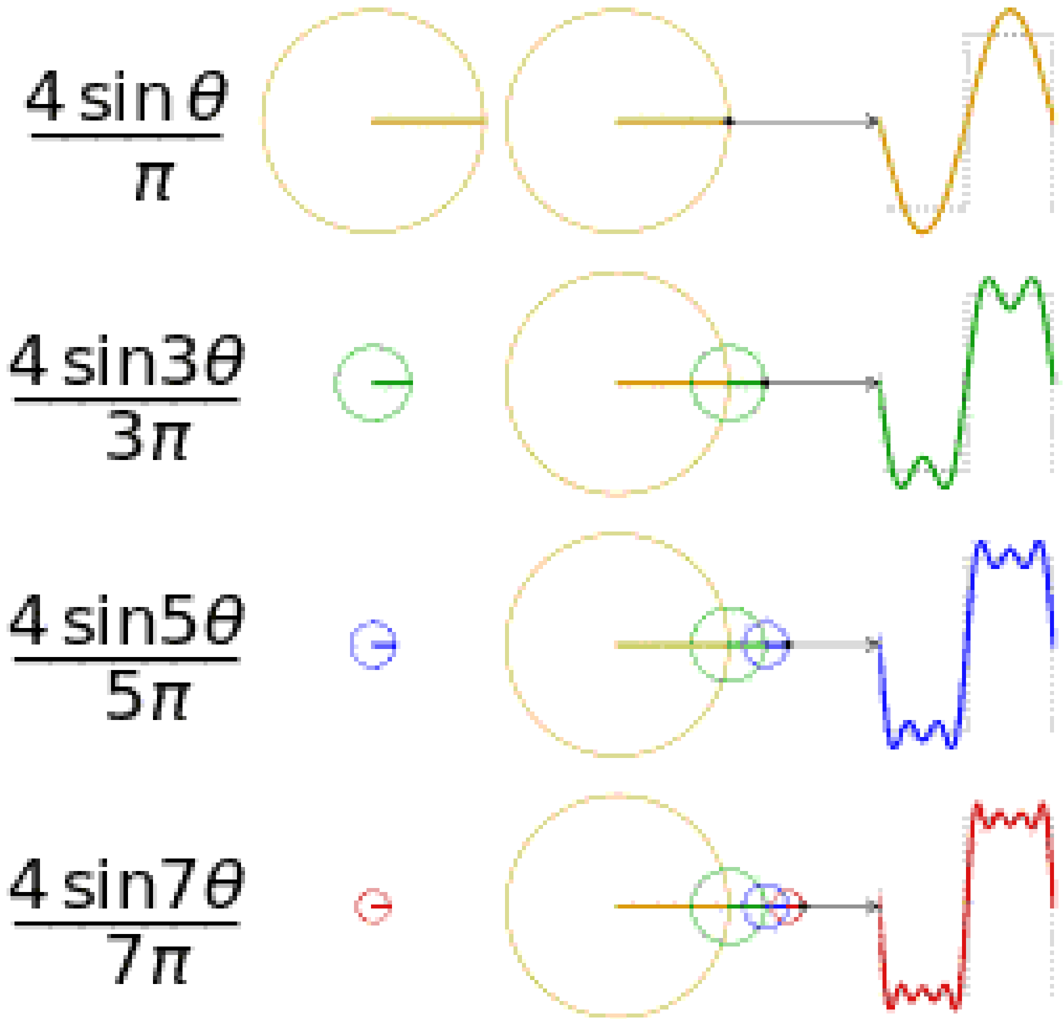


Back to the Euler's formula

Many Frequencies



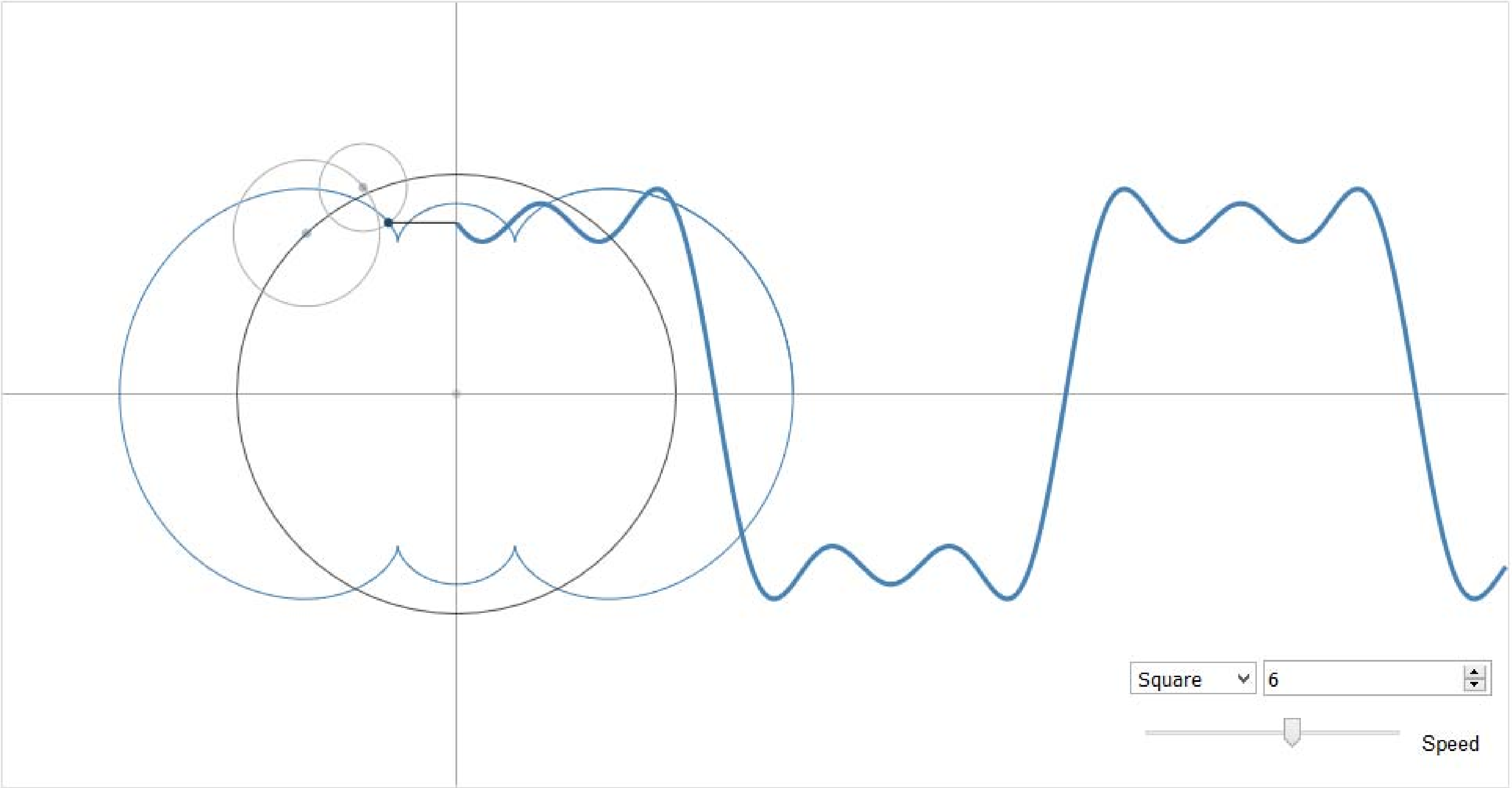
Fourier Series: Ex 2



Fourier Series: Ex 2

[http://codepen.io/anon/pen/jPGJMK/]

Fourier Series visualization

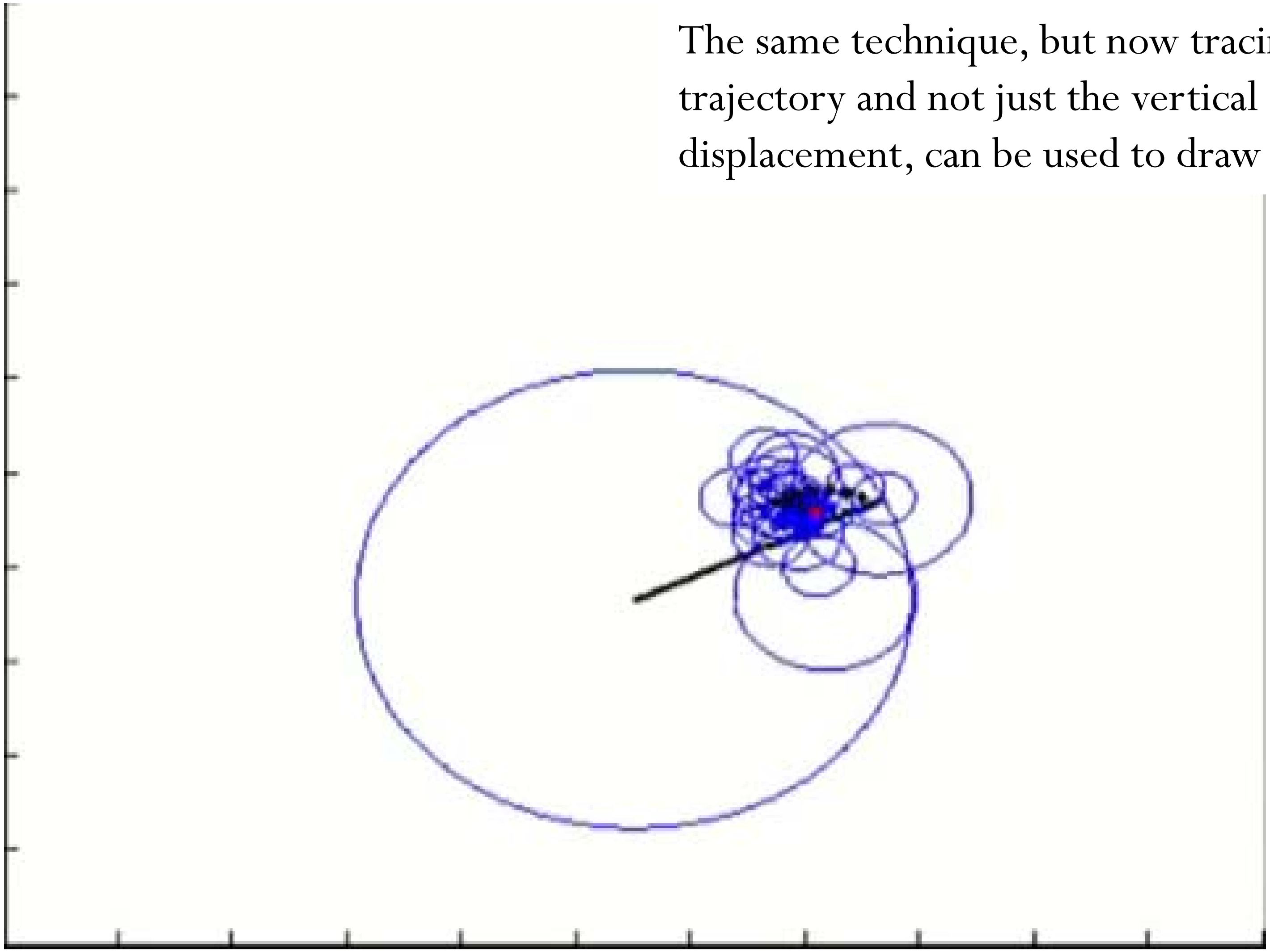


From Wikipedia: Open in a new window.

[http://bl.ocks.org/jinroh/7524988]

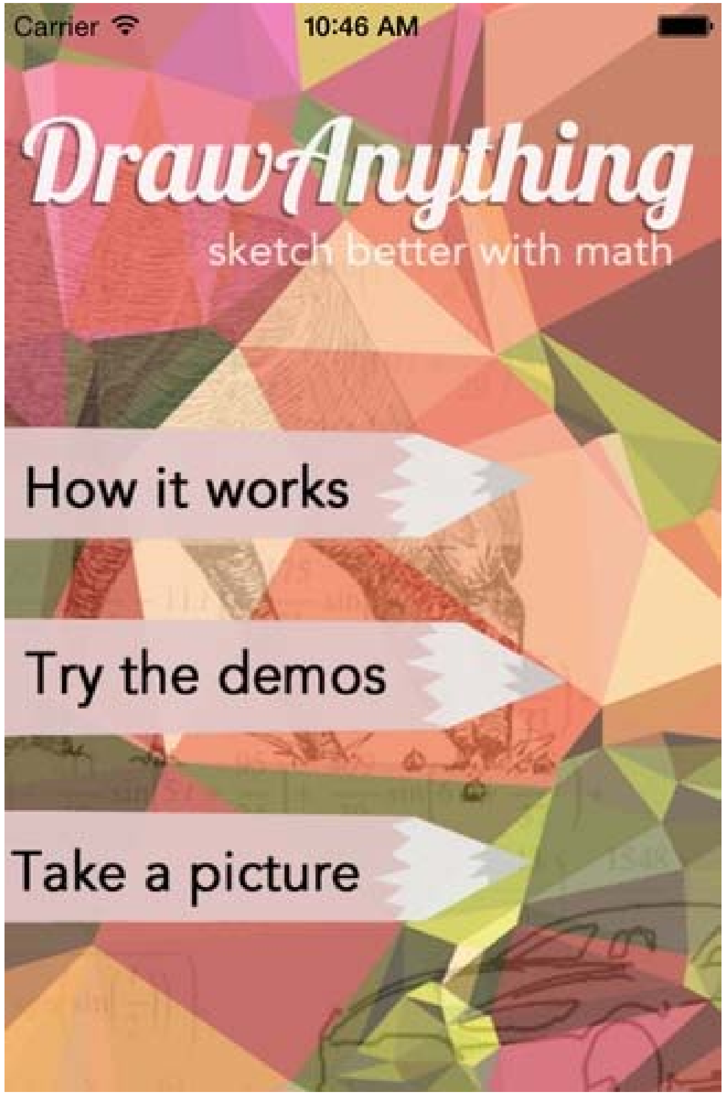
Fourier Series: Drawing

The same technique, but now tracing the whole trajectory and not just the vertical displacement, can be used to draw "anything".

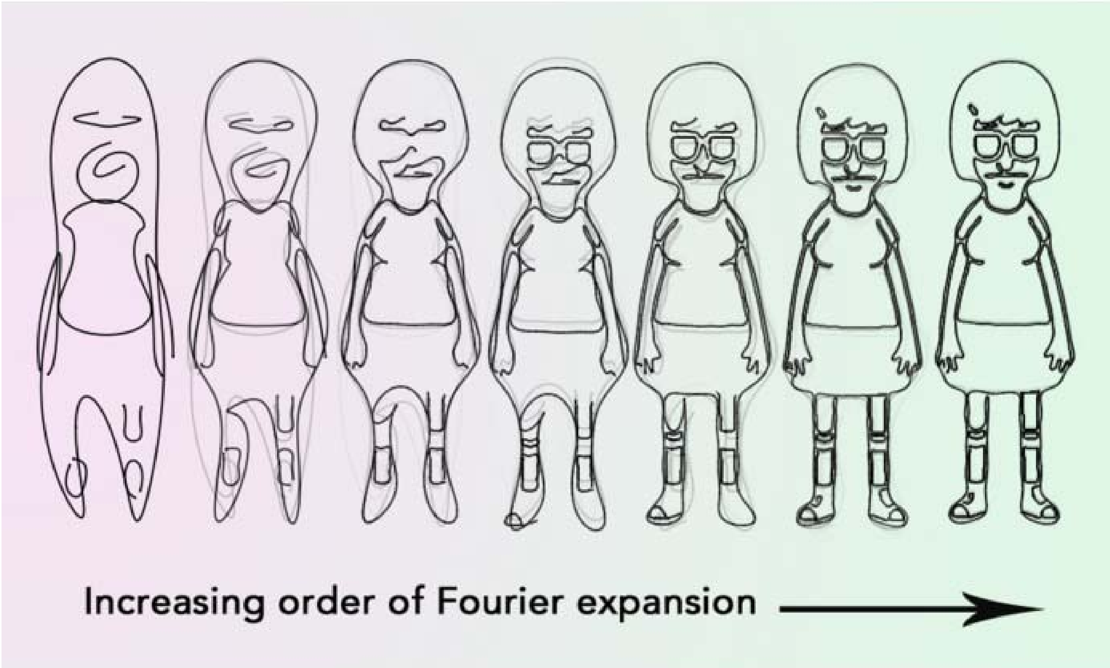


[https://www.youtube.com/watch?v=QVuU2YCwHjw&t=1m]

Fourier Series: Drawing



Draw Anything is an iOS app that harnesses the computational power of the Wolfram Programming Cloud to automatically create step-by-step drawing guides.



[http://devpost.com/software/draw-anything]